Differential Behavior

Introduction
In this article, we will investigate the operation of a differential. It will begin by looking at theory of differential operation, then move into analyzing test data to verify this theory. In February 2010, approximately two weeks of testing was performed at Berta Mostorsports using a heavily instrumented TC2000 car. This testing will form the basis for the analysis.

Background and Operation of a Differential
A differential has two main functions: allowing the driven wheels to rotate at different speeds while applying driving torque. In passenger cars, the most common type is an open differential. In racing cars, it is more common to use some sort of a limited slip differential or spool. For open differentials, the drive torque supplied to the left and right wheel is (approximately) equal. For limited slip differentials and spools, the wheel drive torque can be unequal. The relationship between the two wheel drive torques is what distinguishes the types of differentials.

For clutch-pack limited slip differentials, torque can be transferred directly from the differential housing to the side-gears through the clutch pack, thereby bypassing the differential gear set. The effect of this is to allow a difference in the left and right torque. The difference in drive torque is governed by two interrelated factors: the clutch and the tires. When there is a torque difference that is insufficient to cause the differential clutch to slip, the tires determine the torque difference based on their slip ratio, the vertical load, camber and slip angle. As the torque difference grows, there is a point at which the clutch will begin to slip. Once this occurs, the torque difference cannot grow any further as the clutch friction has transitioned from static to dynamic friction. The maximum difference between the left and the right driving torques is governed by the following relationship (much of the mathematical development presented here is taken from [1]).

\[ T_l - T_r \leq \text{sgn} (\Omega_l - \Omega_r) \cdot T_c \]  

(1)

Where:

\[ T_l = \text{Driving torque, left wheel} \]
\[ T_r = \text{Driving torque, right wheel} \]
\[ \Omega_l = \text{Rotational speed of left wheel} \]
\[ \Omega_r = \text{Rotational speed of right wheel} \]
\[ T_c = \text{Differential clutch torque} \]

Equation 1 indicates that the difference between the left and right wheel drive torques must be less than or equal to the frictional torque of the clutch pack.

There are two main types of clutch-pack differentials. The most common is a Salisbury differential. The other type is a passive-clutch type. For a passive-clutch differential, \( T_c \) is a constant. For a Salisbury differential, \( T_c \) varies with the applied driving torque, \( T_{app} \) (which is given by \( T_{app} = T_l + T_r \)). In general, the clutch torque for a Salisbury differential is given as:

\[ T_c = C \cdot T_{app} + B \]  

(2)

The coefficient \( C \) depends on the ramp angle (the ramp angle is shown in Figure 1), the ramp radius, the clutch effective diameter and the friction properties of the clutch material. The parameter \( B \) indicates the clutch torque preload – how much torque the clutch can resist when no torque is applied to the differential housing. The coefficient \( C \) is often not the same for driving and braking torques applied to the differential. In many racing applications, the ramp angle will be such that \( C \) is nearly zero for braking and quite large for driving torques. It should be noted that the braking torque applied to the differential is only the engine braking torque except in the rare situations where a single inboard brake is used.

Torsen differentials operate in a manner very similar to a Salisbury differential. Usually, the preload term, \( B \), is nearly zero for a Torsen differential. The coefficient \( C \) depends on the design of the gear set.

Spools and open differentials can be thought of as special cases of a Salisbury differential. For a spool, the clutch torque, \( T_c \), is infinite. For an open differential, the clutch torque is zero.

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1Throughout this article, the term clutch refers to the clutch pack inside the differential and not the clutch that connects the engine and transmission.

2In this case the two tires have the same rotational speed, but likely have different effective radii and longitudinal speeds.

3The ramp radius is the distance from the center axis of the differential to the middle of the ramp.
Viscous differentials have a behavior slightly different than that described in Equation 1. The difference in left and right drive torque can be given by:

\[ T_l - T_r = f(\Omega_l - \Omega_r) \]  

(3)

In this equation, \( f() \) is a function that determines how much torque is generated when there is a speed difference of \( \Omega_l - \Omega_r \) between the left and right driveshafts. This function is dependent on the viscous properties of the differential fluid and on the physical design of the insides of the differential. The function may be nearly linear, or it may be non-linear. This details of this function are beyond the scope of this paper.

The differential influences the maximum total drive torque that can be applied by the engine and driveline. For situations where the left and right wheels have different grip, whether due to different surfaces or due to lateral weight transfer, the differential has a large effect on how much total drive torque can be applied. For differentials that obey the behavior described in Equations 1 and 2, the maximum total drive torque is given by the following equation, assuming that the right tire has a lower maximum torque:

\[ T_{\text{app}} = T_{r} + \min \left\{ \left( \frac{1+C}{1-C} \right) \left[ \frac{T_{r}^{\text{max}} + \frac{B}{1+C} \lambda}{\lambda} \right], T_{l}^{\text{max}} \right\} \]  

(4)

Where \( \lambda \) is the ratio of the maximum drive torques of the two wheels. If the right wheel has less grip, \( \lambda = \frac{T_{l}^{\text{max}}}{T_{r}^{\text{max}}} \). Figure 2 shows this relationship for several values of \( C \) and \( B \). In Figure 2, the result of Equation 4 is normalized to the torque at which both tires are at the limit torque \( \left( T_{\text{app}} / (T_{r}^{\text{max}} + T_{l}) \right) \). This is called Normalized Drive Torque in the figure.

As is intuitively expected, differentials with more “lock” (higher values of \( B \) and \( C \)) can utilize the available grip more effectively.

**Instrumentation**

To characterize a differential installed in a vehicle, several measurement variables need to be considered. First, the drive torque applied to each driven wheel need to be measured. This can be done either with instrumented driveshafts or with wheel force transducers such as the Kistler RoaDyn series, shown in Figure 3. Wheel encoders are also required as they are necessary to determine whether the differential is locked or differentiating. These are built into the wheel force transducers. Basic handling sensors like a lateral accelerometers and a yaw-rate gyro are also required. An inertial platform, like the GeneSys ADMA (Figure 4), includes a three-way accelerometer and a three-way gyro and provides the required handling measurements.
Differential Operation
To analyze a differential alone, the best approach is to look at a differential diagram (one is shown in Figure 5). These diagrams show the left drive torque versus the right drive torque. When examining differential diagrams, it is useful to think about them in an alternative coordinate system. It’s useful to think in terms of driving torque and torque difference. Since driving torque is the sum of the left and right drive torques, the driving torque is a line at 45° to the axes. Torque difference is perpendicular to driving torque. These are indicated on Figure 5.

The information that can be obtained from a differential diagram like that shown in Figure 5 is the preload, $B$, and the $C$ coefficient. The preload is the width of the “waist” of the differential diagram. By writing a simple script in GNU Octave or Matlab, the data can be overlaid with a diagram constructed from the equations given in the Background. Figure 6 shows this. For this diagram, the preload is approximately 350Nm and the $C$ coefficient is approximately 0.55.

These differential diagrams show the operating envelope of the differential. To gain more detailed insight into how the differential operates, there are several other graphs that can be used.

By combining Equations 1 and 2 we obtain:

$$T_l - T_r \leq [\text{sgn} (\Omega_l - \Omega_r)] (CT_{app} + B) \quad (5)$$

Figure 3: A wheel force transducer
Figure 4: A GeneSys ADMA Inertial Platform
Figure 5: A differential diagram
Figure 6: A differential diagram showing data and the theoretical boundaries
Figure 7: Torque difference versus speed difference for a constant radius steady-state circular test (50m radius).

By assuming a linear tire and vehicle model, equations for the yaw moment produced in these two regimes were developed in Equation 1 and are shown in Equations 7 and 8. It is important to note that the differential can operate in only one regime at a time.

Which regime is in effect is determined by the torque difference (and hence the yaw moment produced). The maximum value is that in the differentiating regime. If the spool regime yaw moment is less than the differentiating regime yaw moment, the clutch will not slip; if the spool regime yaw moment is greater than the differentiating regime yaw moment, the clutch will begin to slip and the yaw moment will be limited to that of the differentiating regime.

Figure 8 was created using information about the car tested and was produced for a 50 m turn radius.

To read Figure 8, pick a longitudinal force, X. Next, determine which of the two regimes will be in effect at low lateral acceleration. If the yaw moment (and hence torque difference) necessary for spool operation is larger (in magnitude) than the differentiating yaw moment, then the differential will differentiate: the clutch will be slipping. If the yaw moment for spool operation is smaller in magnitude than the yaw moment when differentiating, then the differential will act as a spool. In Figure 8, for X = 2400N, the differential will differentiate until the lateral force exceeds about 9500N. At higher lateral forces, the differential will act as a spool. For the X = 0N case, the differential will always differentiate, regardless of lateral force.

It should be noted here that the tire model used in this simple linear model is a linear model that has a slip stiffness depending on the lateral load, but not coupling effect between longitudinal and lateral characteristics. It may not be possible for the tire to operate at all the points shown in Figure 8 as they may be outside of the friction ellipse of the tire.

Vehicle Dynamics
To investigate the vehicle dynamics effects of the differential, we will look at the yaw moment produced by the differential. This yaw moment is related to the torque difference according to the ratio of the wheel radius to the track width of the driven axle (this assumes that the radius of the tire does not change considerably). This relationship can be given as:

\[ N = \left( \frac{t}{R_t} \right) T_c \]  

Where:

- \( N \) = Yaw moment
- \( t \) = Driven axle track width
- \( R_t \) = Tire radius
- \( T_c \) = The torque difference, as defined above

For a Salisbury differential, there are two operating regimes: the differentiating regime, and the spool regime. In the differentiating regime, the speed of the left and right wheels are different (the clutch is slipping). In the spool regime, the torque difference is insufficient to overcome the clutch frictional torque, so the left and right driveshafts are locked together.

\[ \text{sgn}(x) \] operator is 1 when the argument, \( x \) is positive and −1 when the argument is negative.

Figure 7, which shows the torque difference versus the speed difference for testing performed on a 50m skidpad at low longitudinal force, shows the behavior predicted by Figure 8. There are no data points where the speed difference is zero indicating that the differential is differentiating at all times on to 50m skidpad as predicted by Figure 8. There is significant scatter in the actual torque difference seen in the objective measurement and that predicted by the linear model. This indicates that, as expected, the behavior of the tires are more complex than the linear
\[ N_{\text{spool}} = \left( \frac{2V \rho h Y}{r} - t^2 (F_{z0} + hX / (2w)) \right) \frac{r (X + 2C_x (F_{z0} + hX / (2w)))}{2V (F_{z0} + hX / (2w)) - \rho h Y} - 2C_x \rho h Y \]  

\[ N_{\text{diff}} = -\frac{t T_c}{R_t} = -\frac{t (C X R_t + B)}{R_t} \]

Where:

- \( V \) = Vehicle forward speed
- \( \rho \) = Fraction of weight transfer occurring on the driven axle
- \( h \) = The center of gravity height
- \( Y \) = The lateral force on the vehicle
- \( X \) = The longitudinal force on the vehicle
- \( r \) = The yaw rate
- \( F_{z0} \) = The static load on the driven tires (assumed equal)
- \( w \) = The wheelbase
- \( C_x \) = The slip-stiffness coefficient of the tires

Figure 8: Yaw moment produced by the two operating regimes of the differential on a 50m turn radius at three levels of longitudinal force
model used. Some of the scatter may be due to the dynamic response of the system.

The behavior predicted by Figure 8 can also be seen in Figure 9. Both of these figures show late yaw moment from the differential plotted against the lateral acceleration (or lateral force, which is proportional). In Figure 9, we see that the yaw moment produced by the differential is nearly constant as the lateral acceleration (lateral force) is varied. As seen earlier in Figure 7, the differential is differentiating at all times during this test, so the spool operating regime is not seen in Figure 9.

Figure 10 shows data for several laps of a race track. This data qualitatively supports the model presented here. For higher longitudinal accelerations, the yaw moment increases with the lateral acceleration. This indicates that the differential is acting as a spool at these higher longitudinal accelerations. The behavior seen in Figure 10 can only be compared qualitatively with the behavior shown in Figure 8 because the data was not collected at a constant turn radius.

Conclusion
In this report, we saw how measured differential characteristics compare with a simple mathematical model of a differential. There were good qualitative correlations; to obtain better correlation, a non-linear and dynamic model is probably required. We also used wheel force transducers to determine the characteristics of the differential (the parameter $C$ and $B$).

References