

Designing Steering Geometry

When you're designing steering kinematics, the goal is to orient the tire to the road in the optimal orientation. But, how do you know the optimal orientation? Tire data, of course!

One of the basic decisions when designing a steering system is how much Ackermann you want. The answer to this is determined directly by the tire characteristics, and you can answer this question by using OptimumT in a clever way.

So, let's try it! We will design the steering geometry for a small autocross car (210kg) as an example. The data that we'll use was collected at Calspan by the FSAE TTC (Tire Testing Consortium). Since we're using a race car for the example, our goal is to generate the maximum lateral force from the tires. We'll start by taking raw tire data that was collected on a tire testing machine and import it into OptimumT. Next we'll fit a Pacejka 2002 tire model to this data using the OptimumT Model Fitting Tool. For more information about this process, please refer to the OptimumT documentation. What we end up with is shown in Figure 1.

Once we have a tire model, we can use OptimumT's visualization tools to look at some derived quantities. We can also use the OptimumT Add-In to directly incorporate the tire model into Excel or Matlab. We'll look at the slip angle at which the peak lateral force occurs at different loads and camber angles. This will allow us to design the steering geometry to suit the tire. Before we get to that, however, we need to find the vertical load and camber of each of the front wheels. We'll do this in two steps: first using a simple method, then a more accurate way.

Simplified Steering Geometry

If, for a moment, we ignore the camber of the front tires, we can find the desired Ackermann angle very easily. We need to estimate the maximum lateral acceleration that the car can achieve. We can do this by constructing a simple weight transfer spreadsheet and by using OptimumT. We need to know the mass of the car (for now we'll ignore the non-suspended

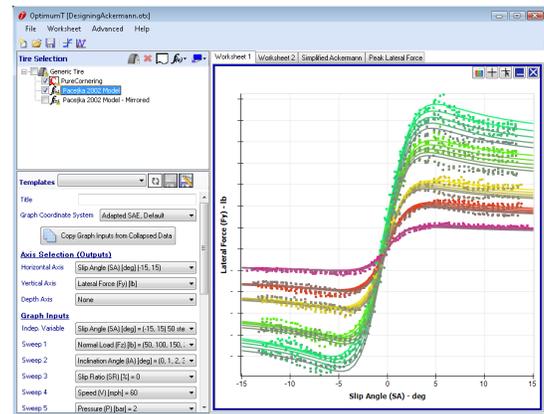


Figure 1: Tire data and model

weight transfer), the CG height, the weight distribution, the front and rear roll stiffnesses and the track.

For this example, the vehicle configuration used is shown in Table 1.

Mass (m)	210kg
Weight Distribution (ρ)	45% front
CG Height (h)	140mm
Front Track (t_f)	1300mm
Front Roll Stiffness (K_f)	220Nm/°
Rear Roll Stiffness (F_r)	200Nm/°

Table 1: Basic vehicle parameters

To calculate the lateral weight transfer, we'll use the following formulas:

$$M_{roll} = A_y m h \quad (1)$$

$$\theta_{roll} = \frac{M_{roll}}{K_f + K_r} \quad (2)$$

$$F_{zr} = 0.5 \rho m g + \frac{\theta_{roll} K_f}{t_f} \quad (3)$$

$$F_{zl} = 0.5 \rho m g - \frac{\theta_{roll} K_f}{t_f} \quad (4)$$

$$A_y = \frac{(F_{yr} + F_{yl})}{\rho m} \quad (5)$$

- A_y is the lateral acceleration
- M_{roll} is the roll moment
- θ_{roll} is the roll angle

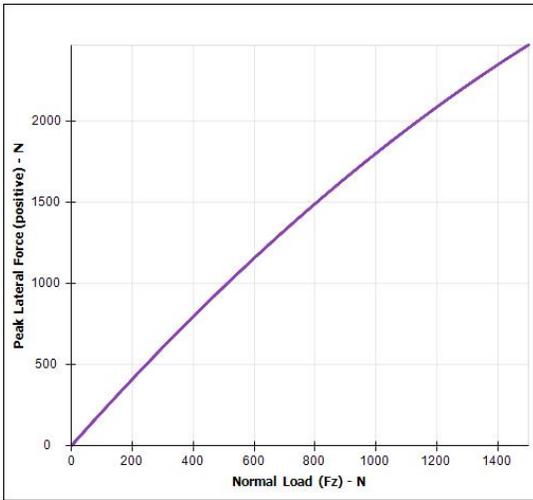


Figure 2: Maximum lateral force versus load

- F_{zr} and F_{zl} are the right and left tire vertical loads
- F_{yr} and F_{yl} are the right and left tire lateral forces
- g is the acceleration due to gravity

We'll perform an iterative calculation to find the maximum lateral acceleration using the equations above. Using Figure 2, we will also use them to find the vertical load on the two front tires. This vertical load will be used to find the ideal slip angle. We find that the right tire has a vertical load of $686N$ and the left tire has a load of $241N$. In these calculations, we implicitly assume that the front axle is the limiting axle; thus the car has terminal understeer.

We can use OptimumT to plot *Ideal Slip Angle versus Normal Load* (Figure 3). By picking out the slip angle at the two vertical loads that we calculated earlier, we find that the outside tire should operate at 4.38° slip angle and the inside tire should operate at 4.11° . This can be used to find the appropriate steering geometry, and indicates that that the car needs positive Ackermann.

More Advanced Steering Geometry

We can improve the calculations by including the camber. We could do this by using graphs created in

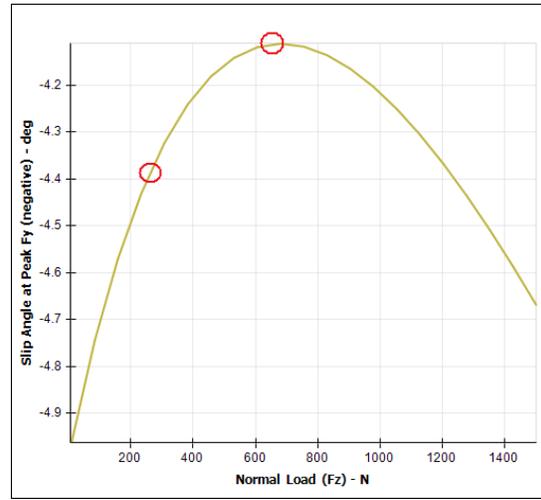


Figure 3: Ideal slip angle versus load

fx = CalculateFy(F3,F1,0,F2,10,10,A10)					
C	D	E	F	G	
		Slip Angle	4	deg	
		Inclination Angle	0	deg	
		Vertical Load	1000	N	
		Lateral Force=	-1802.61	N	

Figure 4: Example of OptimumT Add-In use in Excel

OptimumT, but an easier way is by using the OptimumT Add-In. This allows you to make calculations with OptimumT inside Excel or Matlab. For example, if you want to find the lateral force calculated by a tire model that you created in OptimumT, you can easily do this with the OptimumT Add-In. This is shown in Figure 4.

To find the desired steering geometry, we can create a new spreadsheet in Excel that calculates the lateral weight transfer when both front tires are generating the peak lateral force. The finished spreadsheet is show in figure 5.

The same formulas used in the simplified calculations for the weight transfer are used here. In these calculations, we can use the OptimumT Add-In to find the maximum lateral force that the tire can generate. We use the OptimumT Add-In function:

`CalculateFyPeakNegative()`

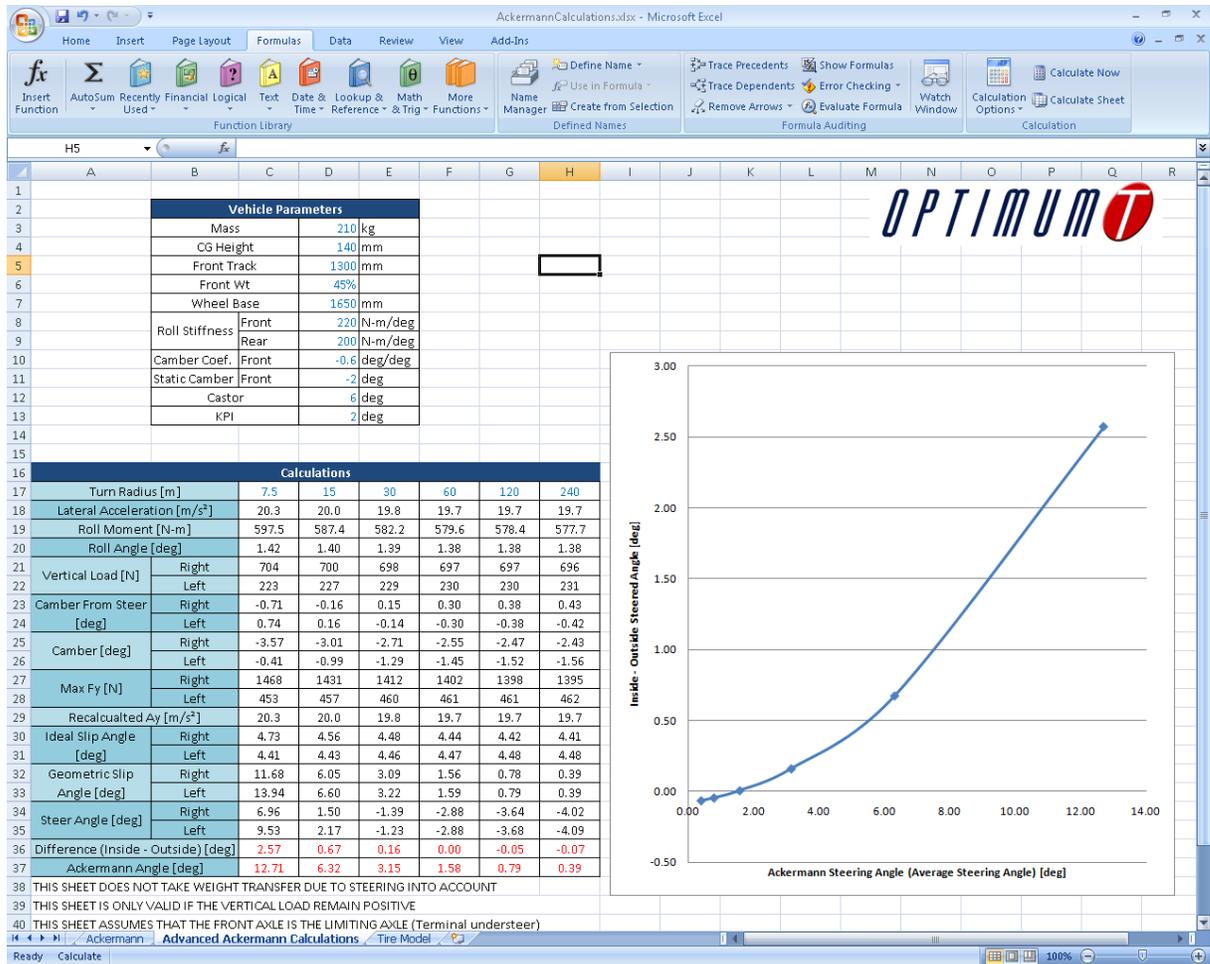


Figure 5: Ackermann design spreadsheet

By using this function in the Excel spreadsheet, we can automate the process of finding the maximum lateral acceleration. We create a "circular reference" in the spreadsheet and Excel will automatically perform an iterative calculation.

To complete the rest of the calculation, we need a little bit more information about the vehicle. For this example, we use the following vehicle parameters. The parameters given earlier still apply.

Wheelbase (l)	1650mm
Camber Coefficient (C)	$-0.6^\circ/^\circ$
Static Camber (γ_0)	-2°
Caster (ϑ)	6°
KPI (λ)	2°

Table 2: Vehicle parameters required for advanced analysis

Once we have these values, we can expand the calculations performed earlier by taking into account the camber of the two steered wheels. Including camber in the calculations increases the accuracy because we include the camber thrust when finding the peak lateral forces. We calculate the inclination angles as follows:

$$\gamma_r = \gamma_{r0} + \lambda(1 - \cos \delta_r) - \vartheta \sin \delta_r + \theta_{roll}C \quad (6)$$

$$\gamma_l = -\gamma_{r0} - \lambda(1 - \cos \delta_l) - \vartheta \sin \delta_l - \theta_{roll}C \quad (7)$$

- γ_r and γ_l are the right and left inclination angles
- δ_r and δ_l are the right and left steered angles (found later)

The slip angles at which the peak lateral force occurs can be calculated using the OptimumT Add-In. It is dependent on vertical load and the inclination angle. The function used is:

`CalculateSAPeakNegative()`

We use geometric relationships to find the slip angles of the front wheels when they are not steered. This is used to calculate the required steered angle of the two front wheels. The geometric slip angle of the two front wheels can be closely approximated with the following expression:

$$\alpha_r^G = \frac{l}{R + \frac{t}{2}} + \beta \quad (8)$$

$$\alpha_l^G = \frac{l}{R - \frac{t}{2}} + \beta \quad (9)$$

- α_r^G and α_l^G are the right and left geometric slip angles
- R is the turn radius
- β is the body slip angle

We find the ideal steered angle of the two steered wheels using the following formula:

$$\delta_r = \alpha_r^G - \alpha_r^i \quad (10)$$

$$\delta_l = \alpha_l^G - \alpha_l^i \quad (11)$$

- α_r^i and α_l^i are the ideal right and left slip angles
- δ_r and δ_l are the required right and left steered angles

Where the ideal slip angle is calculated with the OptimumT Add-In.

Since these equations depend on the turn radius, we repeat them for multiple steer angles. Since we're designing the steering geometry, we're interested in the difference in steered angle between the inside and outside wheel. When we take the difference in left and right steered angle, the body slip angle will cancel out.

We now have the difference in left and right steered angles versus turn radius. We're trying to design steering geometry, though, so we would prefer to plot this versus some steering angle. We'll assume that the car will be close to neutral steer, so the Ackermann steering angle relationship holds ($\delta_{ack} = l/R$). The difference in left and right steered angle plotted against the Ackermann steering angle is shown in Figure 6. This gives us a target curve when we design the steering geometry.

When we read this graph, we find that these tires, fitted on this particular car, need positive Ackermann. The graph also suggests adding about 0.15°

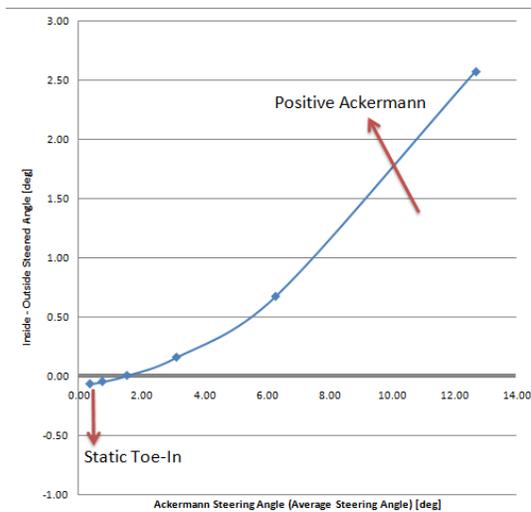


Figure 6: *Ideal steering geometry*

of static toe in (indicated by the offset at zero steering angle). However, the toe will have a large effect on the on-center directional stability of the car. It is best to set the toe angle for handling and stability, rather than for achieving the ideal slip angle during mid-corner.

This tech-tip gives a brief outline of what can be done with OptimumT to assist in designing a steering system. OptimumT and the OptimumT Add-In can be useful tools when evaluating the compromises involved in steering design.

You can easily expand this spreadsheet to give a much better picture of reality. For example, you can include the rear axle in the calculations, you can include the weight transfer due to steering, the non-suspended weight transfer and a multitude of other factors (including the effect on longitudinal forces). You can even create a simple vehicle simulation in Excel or Matlab. This is all made possible through the OptimumT Add-In.

If you have any questions about OptimumT or any of OptimumG's other products and services, please email engineering@optimung.com. Also, don't forget to visit www.optimung.com